

“Exotic” quantum effects in the laboratory?

Ralf Schützhold

Institut für Theoretische Physik, Technische Universität Dresden, D-01062 Dresden, Germany

This Article provides a brief (non-exhaustive) review of some recent developments regarding the theoretical and possibly experimental study of “exotic” quantum effects in the laboratory with special emphasis on cosmological particle creation, Hawking radiation, and the Unruh effect.

I. INTRODUCTION

The combination of quantum field theory with non-trivial classical background configurations – such as a gravitational field – yields many fascinating effects, see, e.g., Birrell & Davies (1982) and Fulling (1989). A striking example is the amplification of omnipresent quantum vacuum fluctuations by the influence of the classical background configuration and thereby their conversion into real particles. This phenomenon may occur in an expanding universe[1] (cosmological particle creation) and lies at the heart of Hawking radiation (Hawking, 1974, 1975), which is related to the Unruh effect (Unruh, 1976) via the principle of equivalence.

In order to discuss these phenomena, it is useful to recall some of the basic properties of the quantum effects under consideration: Since the created particles stem from quantum (vacuum) fluctuations, they vanish in classical limit “ $\hbar \rightarrow 0$ ”. The particles are always created in pairs (squeezed vacuum state) in an entangled state (which is an inherently non-classical feature). In cosmological particle creation, the two particles of each pair have opposite spins and momenta – whereas, for the Hawking and Unruh effects, the two partners occur on either side of the horizon. The two-particle states are pure quantum states while averaging over one partner yields the thermal density matrix (entanglement entropy).

II. THE UNDERLYING ANALOGY

Apart from their quantum nature, which implies a suppression of these effects in a typical laboratory scenario due to the smallness of \hbar , they are intrinsically relativistic phenomena and hence additionally hard to observe in view of the large value of the speed of light c . Therefore, it can be advantageous to consider the analogues of these effects in appropriate condensed-matter systems – where the speed of light c should be replaced by the propagation velocity of the quasi-particles under consideration (e.g., phonons which travel with the speed of sound, see Unruh, 1981). The analogy can be made explicit by writing down the most general linearized $\mathcal{O}(\phi^3)$ low-energy $\mathcal{O}(\partial^3)$ effective action for scalar Goldstone-mode quasi-particles ϕ , which can be cast into the following form

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu \phi)(\partial_\nu \phi)G^{\mu\nu}(\underline{x}) + \mathcal{O}(\phi^3) + \mathcal{O}(\partial^3), \quad (1)$$

where $G^{\mu\nu}(\underline{x})$ denotes some tensor which encodes the details of the underlying condensed-matter system. Via the replacement $G^{\mu\nu} \rightarrow g_{\text{eff}}^{\mu\nu}\sqrt{-g_{\text{eff}}}$, we see that the kinematic aspects (i.e., the equations of motion and the commutation relations etc.) of the quasi-particles ϕ are completely equivalent to a scalar field in a curved space-time described by the effective metric $g_{\text{eff}}^{\mu\nu}$.

Now, how can we exploit this analogy and what can we learn by using it? Of course, the first answer could be that the aforementioned quantum effects have merely been predicted, but not directly observed yet. Hence, the analogy (1) facilitates an experimental verification of these phenomena (by means of their laboratory analogues) and a test of the assumptions/approximations entering their derivation. Furthermore, it allows us to study the corrections imposed on these effects due to the impact of interactions, the back-reaction of the quantum fluctuations onto the classical background, the influence of a finite temperature bath and decoherence etc. Finally, apart from the purely experimental point of view, we may use the condensed-matter analogues as theoretical toy models for quantum gravity which inspire new ideas for studying the influence of the microscopic structure on macroscopic phenomena, for example.

The other way around, the analogy (1) may help us to understand non-equilibrium (quantum) phenomena in general condensed-matter systems in terms of the vast amount of geometrical tools and concepts developed within general relativity and to extract universal features. (For the low-energy quasi-particle kinematics, we may forget about the microscopic details of the underlying condensed-matter system and derive everything from the effective metric $g_{\text{eff}}^{\mu\nu}$, see, e.g., Schützhold, 2008; Schützhold & Uhlmann, 2005; Schützhold & Unruh, 2007).

III. UV CATASTROPHE

Before turning to experiments, let us discuss an example where condensed matter serves as a toy model for theoretical studies. The Hawking effect describes the evaporation of black holes and results in the emission of thermal radiation with the temperature (Hawking, 1974, 1975)

$$T_{\text{Hawking}} = \frac{1}{8\pi M} \frac{\hbar c^3}{G_N k_B}, \quad (2)$$

where M is the mass of the black hole. Hawking's derivation of this effect is based on the semiclassical approach of quantum fields propagating in a classical background space-time and the assumption that this treatment is valid up to arbitrarily large energies. However, there are many reasons to expect that this semiclassical approach fails at high energies, such as the Planck scale. For example, one could argue that it should be impossible to resolve spatial details smaller than the Planck length $\ell_P = \sqrt{\hbar G_N/c^3} \approx 1.6 \times 10^{-35}$ m, because the necessary (Planckian) momentum and hence energy concentrated in such a small volume would collapse into a black hole.

Inspired by the analogy to condensed matter (cf. Jacobson, 1991), a simple model for ultra-high energy deviations from classical general relativity is a modification of the dispersion relation $\omega = ck \rightarrow \omega = \omega(k)$ of the propagating degrees of freedom (e.g., photons, gravitons). E.g., in analogy to special relativity, where the acceleration of a massive particle up to the speed of light is prohibited by a diverging energy $E = mc^2/\sqrt{1 - v^2/c^2}$, one could speculate that quantum gravity impedes a sub-Planckian spatial resolution in a similar manner via $\omega = ck/\sqrt{1 - \ell_P^2 k^2}$. However, calculating the Hawking radiation for such a dispersion relation (Schützhold & Unruh, to appear; see also Unruh & Schützhold, 2005; Brout *et al.*, 1995; Corley, 1998), we find that we reproduce the thermal emission at low energies $\ell_P k \ll 1$, but, in addition, we obtain a large amount of radiation (UV catastrophe) at ultra-high energies $k = \mathcal{O}(1/\ell_P)$. Since black holes are supposed to exist for a macroscopic time, the dispersion relation $\omega = ck/\sqrt{1 - \ell_P^2 k^2}$ should be excluded as a realistic model for ultra-high energy deviations from classical general relativity. One way to avoid this UV catastrophe is a sub-luminal dispersion relation $d\omega/dk < c$.

IV. TRAPPED IONS

For an experimental verification, there are several possible systems. Let us start with trapped ions (see also Schützhold *et al.*, 2008b). In a strongly elongated trap, the (quantized) positions $Q_i(t)$ of the ions satisfy the equation of motion

$$\ddot{Q}_i + \Omega_{\text{ax}}^2(t) Q_i = \Gamma \sum_{j \neq i} \frac{\text{sign}(i-j)}{(Q_i - Q_j)^2}, \quad (3)$$

where Γ encodes the strength of the Coulomb repulsion. If the axial trapping frequency $\Omega_{\text{ax}}(t)$ is time-dependent, the chain of ions expands or contracts linearly, which can be described in terms of the scale parameter $b(t)$. Linearization $\hat{Q}_i(t) = b(t)Q_i^0 + \delta\hat{Q}_i$ around the classical solution $b(t)Q_i^0$ determined by the initial positions Q_i^0 yields the wave equation of the (longitudinal) phonon modes $\delta\hat{Q}_i$

$$\left(\frac{\partial^2}{\partial t^2} + \Omega_{\text{ax}}^2(t) \right) \delta\hat{Q}_i = \frac{1}{b^3(t)} \sum_j M_{ij} \delta\hat{Q}_j \rightarrow \frac{1}{b^3(t)} \nabla^2 \delta\hat{Q}. \quad (4)$$

The time-independent matrix M_{ij} is strongly localized around the diagonal and approaches the second spatial derivative ∇^2 in the continuum limit. Hence, the above equation is analogous to a scalar field within an expanding/contracting universe with $b(t)$ corresponding to the scale parameter of the universe and $\Omega_{\text{ax}}^2(t)$ being roughly analogous to the Ricci curvature scalar.

Consequently, we may simulate cosmological particle creation in an ion trap: Starting in the ground state of the ion chain at rest and subsequently varying the axial trapping frequency $\Omega_{\text{ax}}(t)$ entails the creation of pairs of phonons (where both partner particles occupy the same vibrational mode). The advantage of this set-up lies in the well-developed pumping, cooling, and detection techniques, which are quite advanced in view of the potential of ion traps for quantum computing. As a result, the proposed experiment should be realizable with present-day technology and is presently in preparation. Via the comparison of first and second side-band transitions, it is even possible to test the two-phonon nature of this quantum effect and thereby to distinguish it from classical effects such as heating. As a major draw-back, one should mention the limited size (number of ions), for which a coherent control can be maintained. Therefore, the analogue of cosmological particle creation is most probably feasible with this set-up, but Hawking radiation not yet.

V. BOSE-EINSTEIN CONDENSATES

One option to overcome the size limitation of ion traps – while maintaining low temperatures and coherent control – is a Bose-Einstein condensate (see, e.g., Dalfovo *et al.*, 1999). At length scales much larger than the healing length ξ (typically of order μm), their dynamics can well be described by the Bernoulli equation. Thus, the analogue of a black hole can be realized by a stationary flow which exceeds the speed of sound (of order mm/s) at some point. This point corresponds to the black hole horizon and thus emits Hawking radiation with the temperature being determined by the velocity gradient at that position (Unruh, 1981)

$$T_{\text{Hawking}} = \frac{\hbar}{2\pi k_{\text{B}}} \left| \frac{\partial}{\partial r} (v_0 - c_s) \right|. \quad (5)$$

Inserting the above values, we obtain typical temperatures of several nano-Kelvin, which could also be within reach of present-day technology (Garay *et al.*, 2000; S. Wüster & C. M. Savage, 2007, arXiv:cond-mat/0702045).

Apart from the possibility of achieving very low temperatures, the main advantages of Bose-Einstein condensates are the following: Nowadays, we can handle relatively large samples containing many healing lengths $\xi = \mathcal{O}(\mu\text{m})$ whose shape can be influenced by laser fields, which represent external potentials for the Bernoulli flow. This allows us to generate various velocity profiles for the condensate flow – including quasi-one-dimensional flow –

without the usual problems of supersonic flow velocities (exceeding the Landau critical velocity, cf. Unruh, 2002) induced by the friction with walls etc. The detection of single (or pairs of) phonons is not quite as advanced as for trapped ions, but could also be done in principle via similar optical techniques (see Schützhold, 2006).

Unfortunately, there is also a major draw-back: All the gaseous Bose-Einstein condensates realized in the laboratory are only meta-stable states – the true ground state is a solid. The main decay channels of the gaseous state are inelastic three-body collisions. Such an event transforms three indistinguishable bosons from the cloud into an molecule plus a remaining boson which carries the excess energy/momentum – and thereby all three of them are effectively extracted from the condensate. Now, if all three of these bosons stem from the same macroscopically occupied single-particle wave-function of the condensate ($k = 0$), three-body collisions would just slowly diminish the number of condensed bosons. However, in the presence of inter-particle interactions (which are necessary for the propagation of sound), the many-particle ground state does also contain a small population of the higher single-particle states $k > 0$. This small fraction is called quantum depletion since it is generated by the quantum fluctuations (plus the interaction). Thus, if one of the three bosons involved in the inelastic collision stems from the quantum depletion and is removed, this event causes a deviation from the many-particle ground state – i.e., an excitation (Dziarmaga & Sacha, 2003). Ergo, three-body collisions do also heat up the condensate, which might swamp the Hawking signal to be detected. In order to avoid this problem (and other issues, such as the black hole laser instability, see Corley & Jacobson, 1999), it is probably desirable to employ a very fast detection method.

VI. ELECTRONS IN LASER FIELDS

As the final example, let us consider the Unruh effect (Unruh, 1976). This effect is related (though not equivalent) to Hawking radiation and states that an observer undergoing a uniform acceleration a experiences the Minkowski vacuum as a thermal state with the Unruh temperature

$$T_{\text{Unruh}} = \frac{\hbar}{2\pi k_B c} a. \quad (6)$$

The most direct way of measuring this effect would be to accelerate a detector strong enough and to measure its excitations – which is, however, very hard to do. Thus, we focus on an indirect signature: An accelerated electron would also “see” a thermal bath of photons and hence it could scatter a photon out of this thermal bath into another mode. Translation of this scattering event into the inertial frame corresponds to the emission of two entangled photons by the accelerated electron (Unruh & Wald, 1984), which is a pure quantum

effect. In order to achieve a significant signal, the acceleration a and thus the electric field E accelerating the electron should be very large. Hence, we propose sending an ultra-relativistic electron beam into a counter-propagating laser pulse of high intensity, such that the electric field felt by the electrons is boosted and thus strongly amplified. The probability that one of these electrons emits an Unruh pair of two entangled photons can be estimated via (Schützhold *et al.*, 2006, 2008a)

$$\mathcal{P}_{\text{Unruh}} = \frac{\alpha_{\text{QED}}^2}{(4\pi)^2} \left[\frac{E}{E_S} \right]^2 \times \mathcal{O} \left(\frac{\omega T}{30} \right) \ll 1, \quad (7)$$

where $\alpha_{\text{QED}} \approx 1/137$ is the QED fine-structure constant and $E_S = m^2/q$ denotes the Schwinger limit (Schwinger, 1951). The electric field E , frequency ω , and length T of the laser pulse are measured in the rest frame of the electrons. If $N_e = 10^9$ electrons with a boost factor of $\gamma = 300$ hit an optical laser pulse with 100 cycles, whose (laboratory-frame) intensity is of order 10^{18} W/cm^2 , we obtain around one Unruh event in hundred shots. Since the above values are well within reach of present day-technology, the generation of photons pairs should be feasible[2]. Their detection and the elimination of the background noise is a bit involved, but the unique features of this quantum effect (correlated photon pairs, distinguished phase-space region etc.) should facilitate a measurement.

In order to point out the main advantage of this proposal, one should realize that it is (in contrast to the previous examples in §IV and §V) a *real* relativistic effect, i.e., the speed of light has not been replaced by the speed of sound. Again, the generation of the photon pairs is most probably feasible with present-day technology and their detection should be within reach. The major draw-back lies in the fact that it is not a direct measurement of the (original) Unruh effect: the acceleration is non-uniform and we did not consider the internal excitations of an accelerated detector, but an accelerated scatterer (i.e., electron).

Acknowledgments

This work was supported by the Emmy-Noether Programme of the German Research Foundation (DFG) under grant # SCHU 1557/1-2,3.

Appendix: Classical analogues

It might be illuminating to compare the proposals discussed here to some recent experiments devoted to effects in the vicinity of horizon analogues in the laboratory. These analogues were based on surface waves (cf. Schützhold & Unruh, 2002) in water (G. Rousseaux *et al.*, 2007, arXiv:0711.4767) or helium (Volovik, 2003, 2005) and on light pulses in an optical fibre (T. G. Philbin

et al., 2007, arXiv:0711.4796). First, it should be mentioned that the effects measured in these experiments were not generated by (the analogues of) black-hole horizons, but by other phenomena such as white-hole horizons and/or ergo-regions. The white-hole horizon is the point where the flow velocity drops below the propagation speed of the waves. For surface waves, this point is also known as hydraulic jump (cf. Volovik, 2005). A white-hole horizon does not emit Hawking radiation but entails other classical and quantum instabilities. Second, these experiments could only detect properties of classical[3] wave propagation, i.e., none of the quantum features mentioned in the Introduction (e.g., entangled pairs). Clearly, water waves are most certainly not appropriate for observing quantum phenomena – for superfluid helium and the optical fibres, this remains an open question.

References

- Birrell, N. D. & Davies, P. C. W. 1982 *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge, UK.
- Brout, R., Massar, S., Parentani, R. & Spindel, P. 1995 Hawking Radiation Without Transplanckian Frequencies. *Phys. Rev. D* **52**, 4559.
- Corley, S. 1998 Computing the spectrum of black hole radiation in the presence of high frequency dispersion: An analytical approach. *Phys. Rev. D* **57**, 6280.
- Corley, S. & Jacobson, T. 1999 Black hole lasers. *Phys. Rev. D* **59**, 124011.
- Dalfovo, F., Giorgini, S., Pitaevskii, L. P. & Stringari, S. 1999 Theory of Bose-Einstein condensation in trapped gases. *Rev. Mod. Phys.* **71**, 463.
- Dziarmaga, J. & Sacha, K. 2003 Bose-Einstein-condensate heating by atomic losses. *Phys. Rev. A* **68**, 043607.
- Fulling, S. A. 1989 *Aspects of Quantum Field Theory in Curved Space-Time*, Cambridge University Press, Cambridge, UK.
- Garay, L. J., Anglin, J. R., Cirac, J. I. & Zoller, P. 2000 Sonic Analog of Gravitational Black Holes in Bose-Einstein Condensates. *Phys. Rev. Lett.* **85**, 4643.
- Hawking, S. W. 1974 Black Hole Explosions. *Nature* **248**, 30.
- Hawking, S. W. 1975 Particle Creation by Black Holes. *Commun. Math. Phys.* **43**, 199.
- Jacobson, T. 1991 Black Hole Evaporation and Ultrashort Distances. *Phys. Rev. D* **44**, 1731.
- Schützhold, R. 2006 On the detectability of quantum radiation in Bose-Einstein condensates. *Phys. Rev. Lett.* **97**, 190405.
- Schützhold, R. 2008 Emergent Horizons in the Laboratory. *Class. Quantum Grav.* in press.
- Schützhold, R., Schaller, G. & Habs, D. 2006 Signatures of the Unruh effect from electrons accelerated by ultra-strong laser fields. *Phys. Rev. Lett.* **97**, 121302.
- Schützhold, R., Schaller, G. & Habs, D. 2008 Tabletop creation of entangled multi-keV photon pairs and the Unruh effect. *Phys. Rev. Lett.* in press.
- Schützhold, R., Uhlmann, M., Petersen, L., Schmitz, H., Friedenauer, A. & Schätz, T. 2008 Analogue of cosmological particle creation in an ion trap. *Phys. Rev. Lett.* in press.
- Schützhold, R. & Uhlmann, M. 2005 *Horizon Analogues in the Laboratory*, Proceedings of the Memorial Symposium for Gerhard Soff, Frankfurt, Germany.
- Schützhold, R. & Unruh, W. G. 2002 Gravity wave analogues of black holes. *Phys. Rev. D* **66** 044019.
- Schützhold, R. & Unruh, W. G. (eds.) 2007 *Quantum Analogues: From Phase Transitions to Black Holes & Cosmology*, Springer Lecture Notes in Physics **718**.
- Schwinger, J. 1951 On Gauge Invariance and Vacuum Polarization. *Phys. Rev.* **82**, 664.
- Unruh, W. G. 1976 Notes on Black Hole Evaporation. *Phys. Rev. D* **14**, 870.
- Unruh, W. G. 1981 Experimental Black Hole Evaporation? *Phys. Rev. Lett.* **46**, 1351.
- Unruh, W. G. 2002 Measurability of Dumb Hole Radiation? In *Artificial Black Holes* (eds M. Novello, M. Visser, and G. Volovik) World Scientific, Singapore.
- Unruh, W. G. & Schützhold, R. 2005 On the universality of the Hawking effect. *Phys. Rev. D* **71**, 024028.
- Unruh, W. G. & Wald, R. M. 1984 What Happens When An Accelerating Observer Detects A Rindler Particle. *Phys. Rev. D* **29**, 1047.
- Volovik, G. E. 2003 *The Universe in a Helium Droplet*, Oxford University Press, Oxford, UK.
- Volovik, G. E. 2005 The hydraulic jump as a white hole. *Pisma Zh. Eksp. Teor. Fiz.* **82** 706 (English version *JETP Lett.* **82** 624).

- [1] According to our understanding of the evolution of our Universe, traces of such an effect – i.e., the amplification of quantum vacuum fluctuations during the epoch of cosmic inflation – can still be observed today in the anisotropies of the cosmic microwave background radiation.
- [2] The signal could even be much more pronounced in undulators due to the coherent amplification (constructive interference of many electrons, Schützhold *et al.*, 2008a).
- [3] One of these striking classical phenomena is superradiance, i.e., the amplification of wave-packets in rotating black holes (cf. Schützhold & Unruh, 2002).